16th Symposium on Earthquake Engineering
December 20-22, 2018
IIT Roorkee, India
Paper No. 122

Seismic Vibration Control of Torsionally Coupled Structure using Multiple Tuned Mass Dampers

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Abstract

In the present study, the performance of Multiple Tuned Mass Dampers (MTMDs) to control the earthquake induced response of a torsionally coupled system is investigated. MTMDs with uniformly distributed frequencies are taken for this study. The arrangement of MTMDs is in a row covering the width of the system. A parametric study involving parameters like mass ratio, damping ratio, frequency ratio, normalized eccentricity ratio and the ratio of uncoupled torsional to lateral frequency is carried out for different seismic time history data, in time domain method. Numerical example is taken to find the fruitfulness of MTMDs on reducing both torsional and translational response of the coupled structure. A comparative study is made between torsionally coupled and uncoupled structures. It has been shown that the usefulness of MTMDs in diminishing the lateral response of torsionally coupled system goes up with increase in mass ratio, damping ratio but decreases with an increase in degree of asymmetry.

Introduction

The structural vibrations are effectively reduced by the passive control device, the TMD. The tuning of the damper is done to a specific structural frequency so once that frequency is worked up, the damper will resonate out of phase with the motion by which the structure vibrates and the energy is exhausted by the damper inertia force following up on the structure. The effectiveness of TMD in reducing structural response is decreased by leaps and bounds by mis-tuning of a TMD. Therefore, the use of more than one TMD is recommended so as to boost the effectiveness. It had been shown by Iwanami and Seto[4] in their research that two- TMDs are of much more impact in diminishing the response than a single one. Then, MTMDs with distributed natural frequencies were suggested by Xu and Igusa[8] and additionally worked upon by Yamaguchi and Harnpornchai[5], Abe and Fujino[1], Jangid[6] and Abe and Igusa. From their analysis it had been shown that the MTMDs have benefits over one TMD. This shows that a substantial work is accomplished on the utilization of TMD and MTMDs in suppressing the response of buildings perfect as a two-dimensional model. Even Jangid and Dutta[7] have worked on the impact of MTMDS in managing the torsional response of a system in frequency domain, but very few work has been done to analyze the impact of MTMDs in controlling the torsional response of the system under real earthquake excitation. The present study throws some light here.The objective of the examination are: (1) To delineate the distinct behaviours of torsionally coupled and uncoupled models with MTMDs (2) To study how the frequency bandwidth for torsional and translational responses of a model coupled with torsion differs (3)To investigate the variation of the eccentricity of the primary system, the uncoupled to lateral frequency ratio etc. with the frequency bandwidth of the system model.

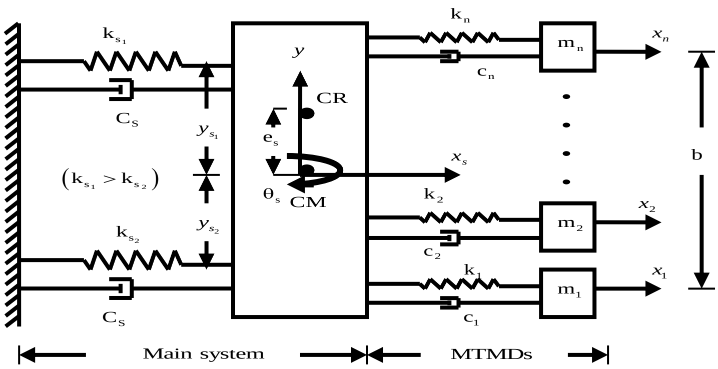
Structural Configuration

The model shows a structural system where *n* TMDs, each having their own dynamic properties, have been placed as depicted in Fig. 1. The structural system which is shown is essentially a system that is combined with torsion. It implies that the CR and CM of the principle system does not coordinate. For this reason, torsional effects are shown by the main system when it is excited in the sidelong way by varied earthquakes.

The breadth *b* of the primary structural system are fully covered by placing the TMDs, equally about the CM. The stiffness and damping ratios are kept constant. Because of this, the CR of TMDs matches with the CM of the primary system. The total mass of the TMDs is found out by adding the masses of all TMDs placed. A lateral force is given at the CM of the main structural system. Consequently, the TMDs and the primary system are vibrated in the horizontal direction. Also, the vibrations due to torsion of the primary system are caused due to the torsional coupling. In this manner, the entire DOF of both the system is *n+2*. The two uncoupled frequency parameters of the primary framework are outlined as

$$\omega_S = \sqrt{\frac{k_S}{m_S}} \tag{1}$$

$$\omega_\theta = \sqrt{\frac{k_\theta}{m_S r_S^2}} \tag{2}$$



Plan view

Figure 1. View of a torsionally coupled system with MTMDs

Where m_s and $k_s = (k_{S1}+k_{s2})$ being the mass and the lateral firmness of the fundamental structural model about the centre of mass, and r_s is the radius of gyration of the principle structural model about the centre of mass;k_{s1} and k_{s2} are the stiffness and y_{s1} and y_{s2} are the distances of the resisting segments from the CM (allude Fig. 1).The eccentricity between the CR and the CM of the primary model is depicted by

$$e_S = \frac{k_{S_1} y_{S_1} - k_{S_2} y_{S_2}}{k_S} \tag{3}$$

The frequencies ω_θ and ω_s may be said to be the natural frequencies of the main system if it were torsionally uncoupled or disconnected, i.e. a system with e_S=0; but however m_s ,k_s and k_θ are equivalent as in the coupled framework. The parameters k_{s1} , k_{s2} and $y_{s1}(=y_{s2})$ are so adjusted that it produces the coveted estimations of ω_s , ω_θ , e_s.. The average frequency of MTMDs is ω_T (i.e. $\omega_T = (1/n)\sum_j \omega_j$). The natural frequency of the j_{th} TMD is given by

$$\omega_j = \omega_T \left[1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right] \tag{4}$$

where the parameter β is the non-dimensional frequency bandwidth of MTMDs defined as

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \tag{5}$$

If k_T and ξ_T are the constant stiffness and damping proportions for every TMDs, at that point the articulation for mass and the damping constant of j_{th} TMD are given underneath

$$m_j = \frac{k_T}{\omega_j^2} \tag{6}$$

$$c_j = 2\xi_T m_j \omega_j \tag{7}$$

The mass ratio is given by the expression as

$$\gamma = \frac{\sum_j mj}{m_s} = \frac{m_T}{m_s} \tag{8}$$

where m_s is the mass of the structure and m_T is the combined mass of MTMDs
The constant stiffness required for each TMDs can be expressed as

$$k_T = \frac{\gamma m_s}{\sum_j 1/\omega_j^2} \tag{9}$$

As the framework coupled with torsion alongside unidirectional eccentricity is defined by two natural frequencies, it is troublesome to outline a tuning proportion for MTMDs as easily as done for an uncoupled SDOF model. So two totally separate tuning frequency ratios are thought of within the paper, specifically

$$f_1 = \frac{\omega_T}{\omega_s} \quad \text{and} \quad f_2 = \frac{\omega_T}{\omega_{s_1}} \tag{10}$$

Where ω_{s_1} is the first natural frequency of the main structural system coupled with torsion.

Equations of Motions

The equations of motion of the model in Fig. 1 are depicted

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{1\}\,f(t) \qquad (11)$$

Where $\{X\}=\{x_s,\theta_s,x_1,x_2,.......,x_n\}^T$ is the displacement vector of the model with x_s and θ_s being the translational and torsional displacement of the main system .The translational displacement for j_{th} TMD is $x_j\,(=1,2....,n)$, $f(t)$ is the horizontal force acting at the CM of the fundamental structural model and $[M]$, $[C]$, $[K]$ are expressed as

$$[M]=diag\left[m_s,m_s r_s^2,m_1,m_2,.....m_n\right] \qquad (12)$$

$$[C]=\begin{bmatrix} c_s+\sum c_j & c_{s\theta}+\sum c_j y_j & -c_1 & -c_2 & \cdots & -c_n \\ & c_\theta+\sum c_j y_j^2 & -c_1 y_1 & -c_2 y_2 & \cdots & -c_n y_n \\ & & c_1 & 0 & \cdots & 0 \\ & & & c_2 & \cdots & 0 \\ & & & & \cdots & \vdots \\ sym & & & & & c_n \end{bmatrix} \qquad (13)$$

$$[K]=\begin{bmatrix} k_s+\sum k_j & k_{s\theta}+\sum k_j y_j & -k_1 & -k_2 & \cdots & -k_n \\ & k_\theta+\sum k_j y_j^2 & -k_1 y_1 & -k_2 y_2 & \cdots & -k_n y_n \\ & & k_1 & 0 & \cdots & 0 \\ & & & k_2 & \cdots & 0 \\ & & & & \cdots & \vdots \\ sym & & & & & k_n \end{bmatrix} \qquad (14)$$

In the above equations c_s, $c_{s\theta}$ and c_θ are the components of the damping matrix of the fundamental model without MTMDs that has been obtained by making an assumption of the modal damping , ξ_s. The coupling term between the translational and the torsional DOF for the fundamental model is denoted by $k_{s\theta}\,(=k_s e_s)$,and the distance of the j_{th} TMD from the CM of the primary system is denoted by y_j.

Numerical Study

A building is shaped as a SDOF structure with TMDs on top of the basic structure as depicted in Figure 1, used to perform the analysis. The characteristics of the basic structure are m_s=2.0×10^5 kg, k_s=7.89×10^6 N/m and damping ratio of structures, ξ_s =2% .The natural time period of the structure, T, is taken as one sec, which is tuned by frequency of the TMDs. The parameters that are held constant are as follows:-

γ=1%; b/r_s=1;The structure is also an asymmetric structure with e_s/r_s not being equal to zero .Here four values of normalized eccentricity ratio(e_s/r_s=0,0.1,0.2,0.3) and three values of ω_θ/ω_s (0.5,1,2)are considered which indicates a torsionally flexible system. Past earthquake ground motion records are selected to perform the analysis, needed, to know the effect of seismic excitation on the performance of TMDs. The structural response quantity which is of utmost interest are the frequency bandwidth of MTMDs. The analysis result is found out for MTMDs which are tuned to the uncoupled lateral frequency of the main structural system and their damping ratio ξ_T is kept at 1%. The ground acceleration time histories of earthquakes taken for this present study are given below

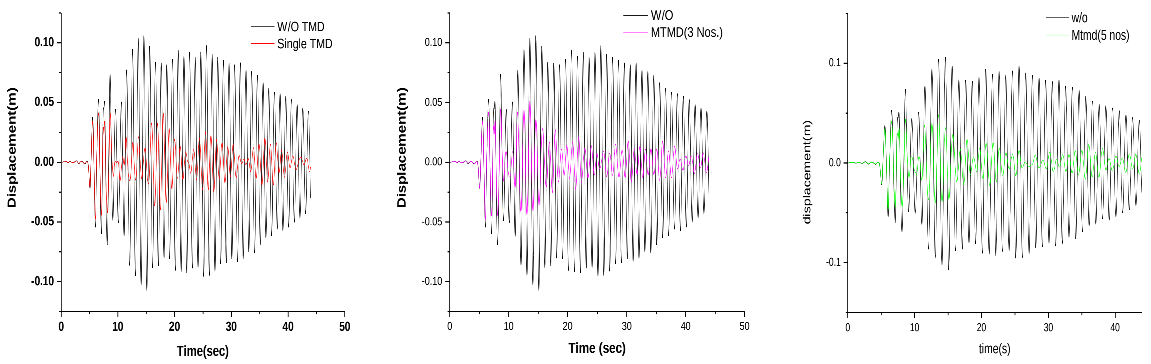
Table 1. Details of selected earthquake time history records

Earthquake	Peak Ground acceleration (PGA)	Duration (sec)	Time Interval between data prints(sec)
Parkfield earthquake (1966)	0.4339g	43.96	0.02
Colinga earthquake (1983)	0.1246g	39.96	0.005

The variation of translational displacements and accelerations of structures with time considering time history data of antecedently chosen earthquakes of Park field earthquake with and without TMDs have been shown in Figure 2 to Figure 5, which gives the clear picture of the impact of the damper in reducing peak structural responses

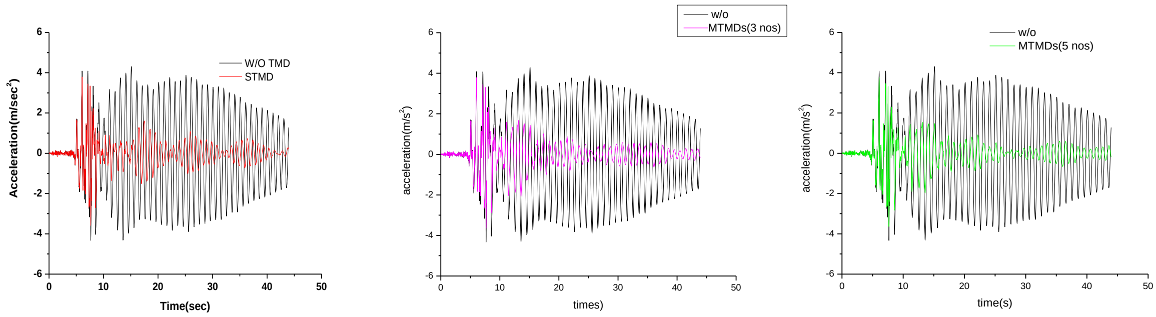
The results show that the fruitfulness of both STMD and MTMDs to cut back the structural displacement is quite substantial for a few earthquake records and for some records the dampers aren't that much impactful, i.e. for Parkfield Earthquake the reductions in displacements and accelerations for STMD and MTMDs are significant.

The peak linear structural displacement reduction for STMD here is 49.3% whereas for MTMDs (3 nos) and MTMDs (5 nos) it is 54.37% and 54.75% respectively. The results are obtained for e_s/r_s=0.1 and w_θ/w_s=0.5(i.e. a torsionally flexible system).The results also shows that an increase in number of dampers increases the reduction percentage.



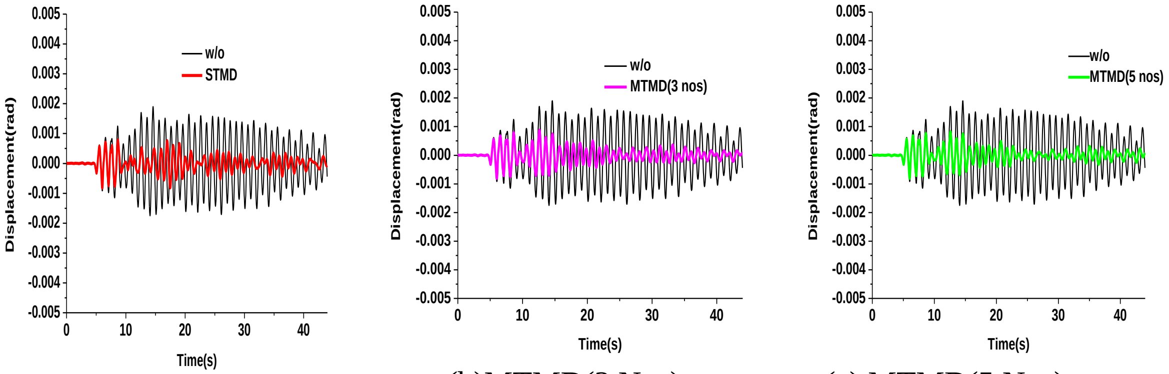
(a) (b) (c)

Figure 2. Variations of translational displacement of structures using (a) Single TMD (b) MTMDs(3 Nos.) and (c) MTMDs (5 Nos) under Parkfield Earthquake



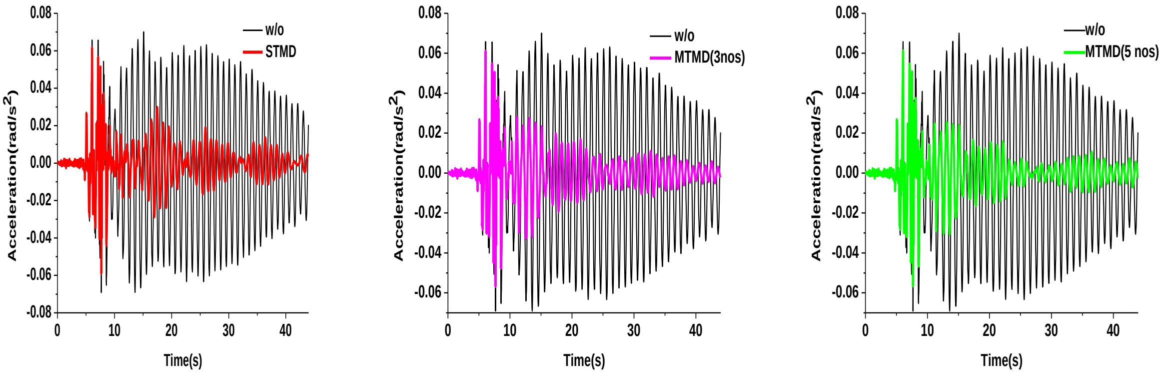
(a) (b) (c)

Figure 3. Variations of translational accelerations of structures using (a) Single TMD (b) MTMDs (3 Nos.) and (c) MTMDs (5 Nos) under Parkfield Earthquake



(a) STMD (b)MTMD(3 Nos) (c) MTMD(5 Nos)

Figure 4. Variations of torsional displacement of structures using (a) STMD (b) MTMDs(3 Nos.) and (c) MTMDs (5 Nos) under Parkfield earthquake



(a) STMD (b)MTMD(3 Nos) (c) MTMD(5 Nos)

Figure 5. Variations of torsional acceleration of structures using (a) STMD (b) MTMDs(3 Nos.) and (c) MTMDs (5 Nos) under Parkfield earthquake

Parametric Study

In Figure 6 the variation of the peak displacement for translation and torsion considering three TMD is plotted against the frequency bandwidth β for ω_θ/ω_S=1.The responses increases with the increase of eccentricity. This gives the obvious fact that the MTMDs should be designed for asymmetric structures taking definite notice into the impact of torsional coupling, otherwise the fruitfulness of MTMDs are overestimated. It is seen that with the increase of β the peak responses increases under real earthquake excitation. Moreover, it can be seen that the impact of MTMDs in suppressing both translational and torsional responses decreases as the eccentricity ratio is increased.

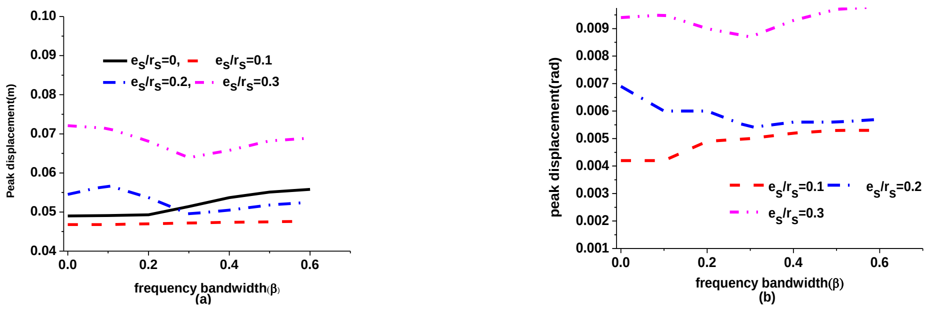


Figure 6. Variations of peak response structures with the frequency bandwidth for ω_θ/ω_S=1,ξ_T=1% and f=1 (a)for translation and (b) for torsion under Parkfield earthquake

The variation of β with the corresponding eccentricity ratio is given in the above table and also the peak translational and torsional responses are shown. Figure 7 depicts the same variation for torsionally flexible structure i.e. for ω_θ/ω_S=0.5. It is shown in the graphs that the MTMDs fares well for torsionally flexible structures than that of the structure with intermediate torsional stiffness. With the increase of eccentricity the response decreases and the effectiveness of MTMDs increases. For torsionally flexible structures the trend is completely reverse to the trend observed for the case of the structure with intermediate torsional stiffness(e_s/r_s=1).One very obvious reason for this might be accredited to the spacing of natural frequencies.

Table 2. Frequency bandwidth and effectiveness of MTMDs (ω_θ/ω_S=1,ξ_T=1%,tuning ratio f=1)

β / e_s/r_s		0	0.1	0.2	0.3	0.4	0.5	0.6
0	$x_{s(\mathrm{m})}$	0.049	0.0491	0.0493	0.0514	0.0537	0.0551	0.0558
	θ_s (rad)	-	-	-	-	-	-	-
0.1	x_s	0.0468	0.0468	0.047	0.0472	0.0474	0.0475	0.0477
	θ_s	0.0042	0.0042	0.0049	0.005	0.0052	0.0053	0.0053
0.2	x_s	0.0545	0.0568	0.0537	0.0495	0.0505	0.0518	0.0525
	θ_s	0.0069	0.006	0.006	0.0054	0.0056	0.0056	0.0057
0.3	x_s	0.0721	0.0714	0.0681	0.0639	0.0658	0.0682	0.0689
	θ_s	0.0094	0.0095	0.009	0.0087	0.0093	0.0097	0.0098

For smaller e_s/r_s , it echoes the same as that found out by Jangid and Dutta(1997) "the natural frequencies are closely spaced and hence the contribution of the mode other than the one which

is being suppressed, significantly influences the response". The variation of β with the corresponding eccentricity ratio is given in the Table 3 and also the peak translational and torsional responses are shown. It is seen that the suppression of torsional response of structures is good and as the MTMDs are tuned to the first mode of vibration which has significant torsion and so the reduction of torsion due to the MTMDs is also significant.

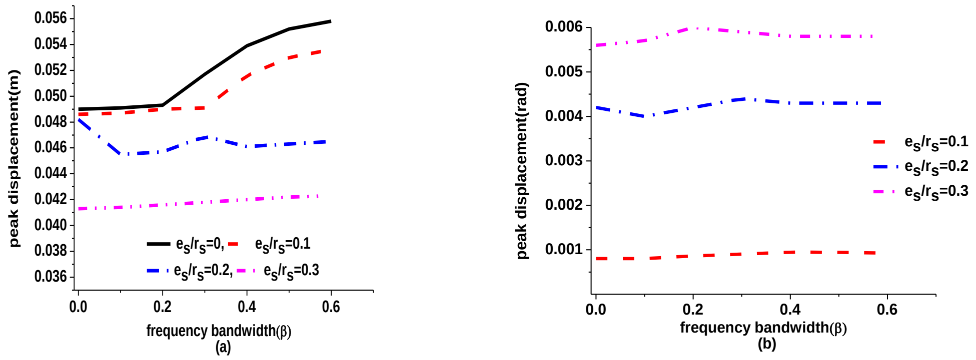


Figure 7. Variations of peak response structures with the frequency bandwidth for ω_θ/ω_s=0.5,ξ_T=1% and f=1 (a)for translation and (b) for torsion under Parkfield earthquake.

Furthermore, from Table-2 and Table-3 it has been shown that the curtailment of translational response of structure due to MTMDs is more for ω_θ/ω_s=0.5 than it is for ω_θ/ω_s=1.The reason for this is that for a given value of e_s/r_s,, the system gets more torsionally coupled for the latter frequency ratio and so the impact of MTMDs become less for ω_θ/ω_s=1.

Figure 8. shows similar varieties for ω_θ/ω_s=2, which means a torsionally hardened framework with distinct lateral and torsional frequencies. The peak displacement for the translational and torsional responses varies in small amount with the frequency bandwidth, for every values of e_s/r_s, and it follows the same trend as that of ω_θ/ω_s=1.The effectiveness of MTMDs does not change by a significant amount for all values of e_s/r_s,

Table 3. Frequency bandwidth and effectiveness of MTMDs (ω_θ/ω_s=0.5,ξ_T=1%,tuning ratio f=1)

β / e_s/r_s		0	0.1	0.2	0.3	0.4	0.5	0.6
0	x_s	0.049	0.0491	0.0493	0.0517	0.0539	0.0552	0.0558
	θ_s	-	-	-	-	-	-	-
0.1	x_s	0.0486	0.0487	0.0490	0.0491	0.0516	0.053	0.0536
	θ_s	0.000801	0.000803	0.000864	0.000902	0.000948	0.000945	0.000926
0.2	x_s	0.0482	0.0455	0.0457	0.0469	0.0461	0.0463	0.0465
	θ_s	0.0042	0.004	0.0042	0.0044	0.0043	0.0043	0.0043
0.3	x_s	0.0413	0.0414	0.0416	0.0418	0.0420	0.0422	0.0423
	θ_s	0.0056	0.0057	0.006	0.0059	0.0058	0.0058	0.0058

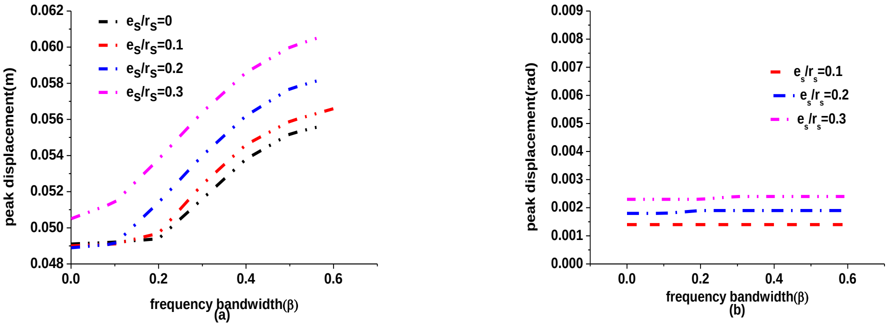


Figure 8. Variations of peak response structures with the frequency bandwidth for ω_θ/ω_s=2, ξ_T=1% and *f*=1 (a)for translation and (b) for torsion under Parkfield earthquake

Effect of Mass Ratio

Figure 9 shows the variation of mass ratio with peak displacement of structure. It is evident from Figure 9 that TMD with higher mass ratio is more effective in suppressing the structural displacement. Here in the figure below (Fig 9) we can see that the peak response of the structure decreases with the increase of mass ratio of structure for both STMD and MTMD.

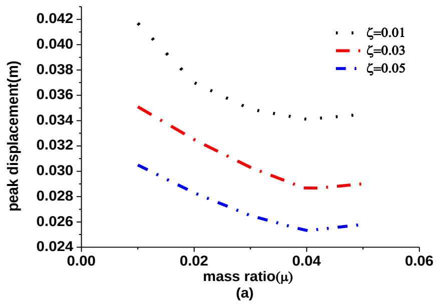


Figure 9. The influence of mass ratio on the maximum translational displacement of structure for different damping ratio for time history data of Colinga earthquake (1966) for MTMD(3 nos)

Effect of Tuning Ratio

Figure 10 shows the variation of tuning ratio with peak displacements for different structures.

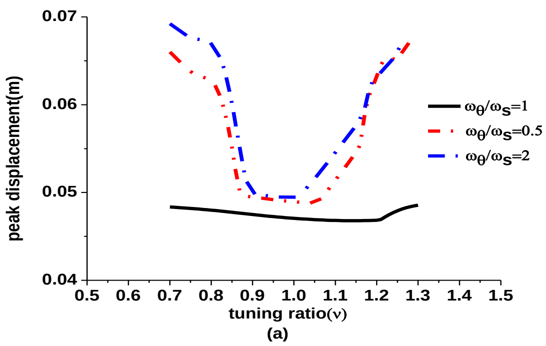


Figure 10. The effect of tuning ratio on the maximum translational displacement of structure for varied damping ratio for history data of Colinga earthquake (1966)for MTMD(3 nos)

The study is conducted for three different structures namely torsionally flexible structures, torsionally intermediate structures and torsionally stiff structures. From this figure it can be be deduced that no specific value is of tuning frequency is there due to which maximum suppression of response for asymmetric systems occurs. It is dependent upon the ratios of ω_θ/ω_s and e_s/r_s. But it is a definite possibility that a tuning frequency can be obtained for a specific combination of ω_θ/ω_s and e_s/r_s for which the response of the asymmetric structure would be minimum. An optimization program can be used to achieve this.

Conclusions

A study of the fruitfulness of MTMDs for the response control of a system coupled with torsion, for real earthquakes has been conducted. An easy unidirectional eccentric model having 2-DOFis supplied with MTMDs. The results cause the subsequent conclusions:

- Usefulness of MTMDs in subduing the translational response is slightly less than that of the corresponding symmetric systems though in some cases(ω_θ/ω_s=0.5) its effectiveness increases than that of the corresponding symmetric systems.
- The frequency bandwidth of MTMDs corresponding to maximum reduction in response of a system changes as the eccentricity of the asymmetric system is changed. So without paying no attention to the impact of torsional coupling, if that frequency of MTMDs is computed, then it might not effectively control the response.
- For torsionally coupled structures the MTMDs are of much more impact than STMD.s But the effectiveness of MTMDs decreases with the increase in eccentricity of the system. However, the case is just opposite for torsionally flexible structures.
- The translational response of an asymmetric system considering torsional coupling decreases with the increase in mass ratio of the MTMDs.
- Tuning frequency of MTMDs for which paramount response control is accomplished for asymmetric model mainly depends upto what extent the system is asymmetric. ($\omega_\theta/\omega_s$and e_s/r_s).

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